Modelling and Control of the Moisture in a Test Bench Flow with Time-delay

Miguel A. Hernandez Perez, Emmanuel Witrant, Olivier Sename

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***Abstract*— Moisture control in systems with time delay is studied in this work to be assessed in a process-control system (Test bench). To further investigate the phenomenon of transport delay in flows, the test bench system has been studied. In this work it is presented the design and validation of a model which describes the dynamics of mass transport phenomena. In order to control the moisture in the test bench, it is design a state-feedback controller such that the closed-loop system is robustly stable has an upper bound for the time delay.**

I. INTRODUCTION

The ability to manipulate flow properties (density, concentration, pressure, etc.) to improve efficiency and performance in the transport of materials is of immense technological importance and it is currently an active research topic in control engineering. Flow control is often encountered in industrial and commercial applications, such as hydraulic networks [1], gas flow in pipelines [2] and flow regulation in mining [3]. The air must be regulated to ensure the proper operating conditions such as temperature or moisture. Flow control can also be applied to the automotive industry, for instance the regulation of the quantity of fresh air in the intake manifold of the engine with exhaust gas recirculation, which is a critical factor for reducing emissions [4].

Flow control problem is often modeled by Partial Differential Equations (PDE), for which usually the reduction and discretization are used as strategy to handle this problem such as shown in [5] and [6]. In [7] an approximation is not considered to solve the hyperbolic systems with n rightward convecting transport PDEs. Using the information from the boundary control and the boundary conditions, this problem is simplified by Lyapunov based techniques. Another approach consists of approximating the transport phenomena by a time delay model, which includes distributed delays, as in [8], where the time-delay approach is derived by using the method of characteristics to express the PDE model as a Functional Differential Equations (FDE) with a distributed delay kernel.

Flow transport is a phenomenon that induces time-delay. This time-delay is caused by the path that crosses the mass in the pipeline. In closed-loop this problem is more difficult to handle. The existence of time delays may result in instability, oscillation and poor performance of the control system. Therefore, lots of efforts have been done in the study of time-delay systems during the last decades, and a great number of results on analysis and synthesis related to time-delay systems have been reported in the literature, as presented in [9], [10], [11], among others. In addition, many different control methods have been proposed in the literature to stabilize uncertain control system with time delay [12], [13], [14].One approach to deal with this problem is to guaranteed cost control, which is concerned with the design of controllers such that the closed-loop system meets the performance objectives and is robustly stable such as in [15], [16].

This paper is concerned with the modeling and control of the moisture in a test-bench available at the University Joseph Fourier, Grenoble. The test-bench, represented in the Figure 1, consists of a heating column encasing a resistor, a tube, two fans, a wind speed meter as well as distributed moisture sensors. Flow control in test bench regulates a Poiseuille flow by the resistance power, fan rotational velocity and the generation of mist inside of the tube. The fan creates an air flow into the tube, the air moves inside of the tube and it is mixed with a mist injection which generates a change of moisture along the tube. The contribution of this paper is to propose a complete model and control methodology to deal the inherent time-delay in the dynamical behavior of the moisture inside of the tube, as well as the variation of the operating conditions in terms of temperature and fan rotational velocity. First a model for moisture regulation in the test bench is obtained. The test bench is modeled as a linear system with parameter uncertainties and time-delay. In order to minimize the control strategy of this kind of systems, from [17] we apply a state-feedback controller and integral-quadratic cost function such that the resulting closed-loop system is robustly stable. The sufficient conditions for stability are presented in a linear matrix inequality (LMI) framework. Finally, to improve the tracking performance, a state-feedback integral control is developed and compared with a classical PI control, using simulation and experimental results on the test bench.

The paper is organized as follows. Section II describes the test bench identification and presents its the model to obtain a moisture control. Section III exposes the problem formulation developed to reach a model with parameter uncertainties and time-delay . Section IV present the problem solution and finally, in the Section V, the control strategy is applied for the regulation of the moisture in the test bench.

Grenoble Images Parole Signal Automatique (GIPSA-Lab), Universite Joseph Fourier, 11 rue des Mathematiques, Saint Martin d'Heres, Grenoble, France. miguel-angel.hernandez-perez, olivier.sename, emmanuel.witrant@gipsa-lab.grenoble-inp.fr

II. MODELING AND IDENTIFICATION

A. Test bench Identification

To further investigate the phenomenon of mass transport in Poiseuille flow and to obtain a model which represents the dynamics of moisture in the system, several experiments were carried out on the test bench. The description of the test bench is presented in Figure 1.

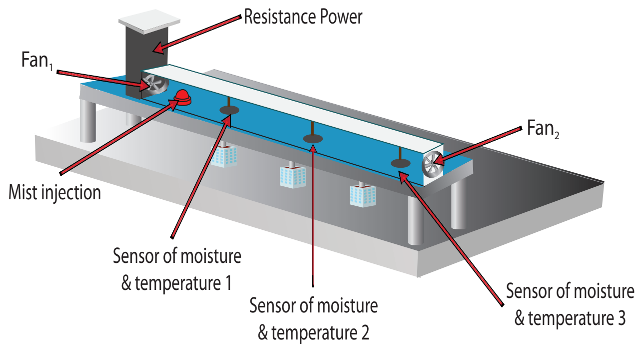

Fig. 1. *Test bench system.*

This device mainly consists of two parts:

1) Sensors:
   - 3 Moisture sensors distributed along the tube.
   - 3 Temperature sensors distributed along the tube.
   - 1 Wind speed sensor.
2) Actuators:
   - 1 Mist injection into the tube (Mist Control).
   - 1 Heater (Resistance power).
   - 2 Fans for circulating the air (Fan rotational velocity).

The objective of the plant is to illustrate the mass transport (concentration between the air flow inside of the tube and the mist injection) along of the tube, i.e., how the time-delay appears when a controller output (mist injection) signal is issued until when the measured process variable (moisture) first begins to respond.

In this section, an identification procedure has been conceived in order to obtain the control-oriented model. Considering the measurement on sensor 3, we can say that there exists a time delay between the measurement of output moisture and the action of the input control (mist injection) in the device.

Moreover such a plant owns large variations of the dynamical behavior according to the changes of heating and wind conditions. We will then analyze the system performance on the whole range and deduces an uncertain control-oriented model.

As first step, It is important to know how the moisture dynamics is affected by temperature and wind speed. To carry out this experiment, the operating condition is taking into account that mist injection is working at 100%, resistance power at 0% and the rotational velocity of the fan at40%

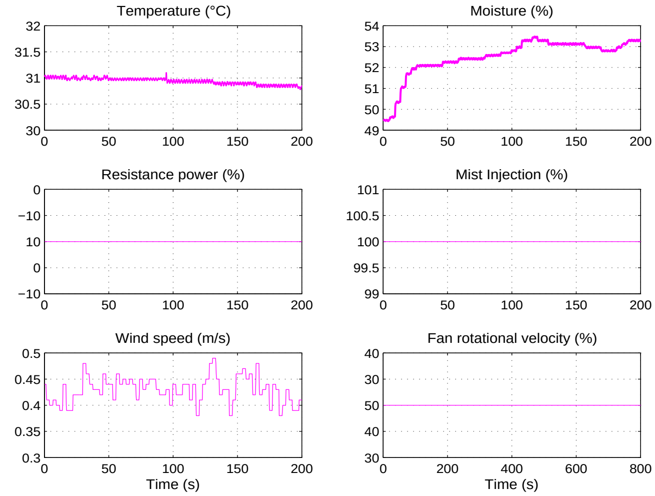

Fig. 2. *Test Bench response when mist injection, resistance power and fan rotational velocity are set to* 100%*,* 0% *and* 40%*, respectively*

Figure 2 shows that the temperature does not change significantly (from $31°C$ to $30.8°C$) when it is applied a mist injection at 100%, velocity remains constant $(0.4m/s)$ and the maximum value of moisture is of 53%.

Now, the objective of these experiments is obtain a nominal model and extreme models of the dynamics of the moisture into the test bench for uncertainties modeling, it is necessary to carry out a lot of experiments taking into account variations in mist injection, resistance power and in rotational velocity of the fan. Hence to reach this model, the next stages are proposed.

1) The first stage of the experiment consists in varying mist injection from 20% to 100%, taking into account that there is no change in the resistance power neither in the fan rotational velocity.
2) The second stage of the experiment consists in increasing the resistance value from 0% up to 100% in intervals of 20% in order to observe the maximum change in the moisture (considering that there is no change in the fan rotational velocity).
3) The third stage of the experiment is an increment in the fan rotational velocity from 20% up to 100% by intervals of 20% in 20%.

Table I shows the parameter variations considered (mist injection, resistance power and fan rotational velocity) to developed the experiments in test bench to obtain the parameter identification of the device.

Remark 1: All increments in parameter variations were done starting from 0% to 100% by intervals of 20%.

To give an overview of the performance of the plant, Figure 3 shows the first stage of the experiments when the operating conditions are: mist injection starting from 0% to 100% with increments of 20%, resistance power at 0% and fan rotational velocity at 40%. From this figure, it is possible to observe that changes in the moisture (with initial condition

TABLE I

TABLE OF EXPERIMENTS

Resis-tance	Mist injection	Fan rotational velocity 20%	40%	60%	80%	100%
0%	$20-100\%$	✓	✓	✓	✓	✓
20%	$20-100\%$	✓	✓	✓	✓	✓
40%	$20-100\%$	✓	✓	✓	✓	✓
60%	$20-100\%$	✓	✓	✓	✓	✓
80%	$20-100\%$	✓	✓	✓	✓	✓
100%	$20-100\%$	✓	✓	✓	✓	✓

in 43.4%) are significant in relation to the temperature when the fan rotational velocity is slow (approximately between 10% and 40%) since the ambient temperature varies from $25.9°C$ to $26.4°C$.

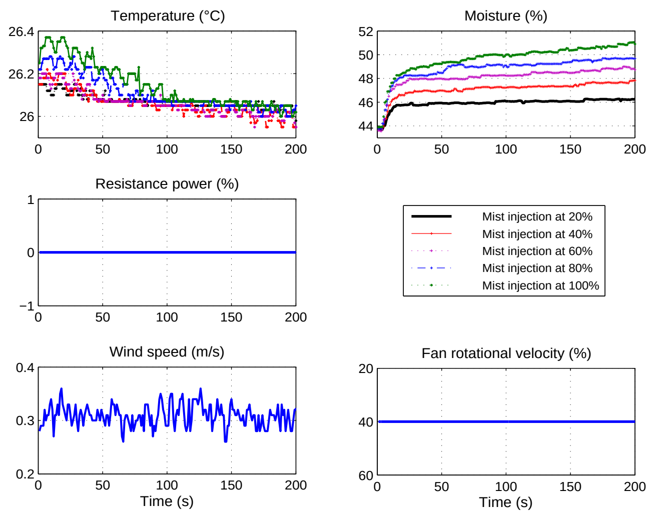

Fig. 3. *Test Bench response with different values in mist input from* 0% *to* 100% *in intervals of* 20%.

The second and third stage of the experiments are show in Figure 4 where it is illustrated the permissible range in which the dynamics of the moisture works. From the Figure 4 it is possible to observe that the temperature does not vary significantly from $28°C$ to $31°C$, while the permissible range of moisture is from 39% to 56% and fan rotational velocity is $0.2s$ approximately.

B. Test bench Model

As shown in II-A, test bench has strong dynamical variations according to the operating point (mist injection, resistance power and fan rotational velocity). To get a simplified control oriented model a first-order time-delay system has been considered which captures the main dynamics of the moisture in the plant.

To obtain the uncertain time-delay model of the device illustrated in Figure 1, all system responses gathered from the above experiments have been considered. Thus, it is possible to enclose the variations of the mist injection, resistance power and fan rotational velocity in 3 parameters (gain, time

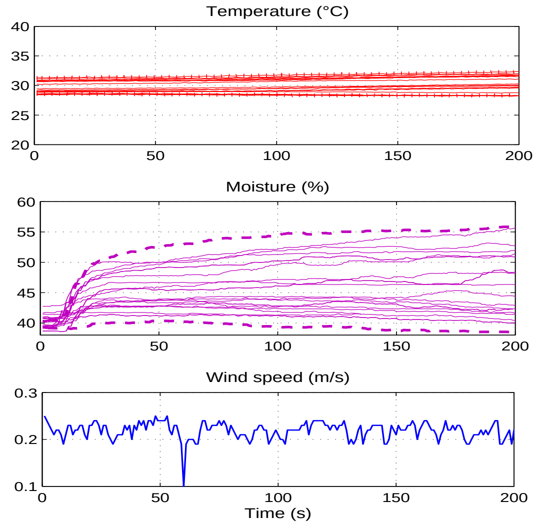

Fig. 4. *Operating range of the moisture dynamics in the test bench.*

constant and time-delay) to generate a first-order transfer function with time delay, which represents the dynamic relationship between the mist injection (control input) and moisture output.

Considering that for each of the previous experiments the dynamics of the moisture are represented by the following transfer function

$$\frac{Moisture(\%)}{MistControl(\%)} = \frac{K}{1+T_p s}e^{-\tau s}, \tag{1}$$

- K= Gain
- T_p= Time constant
- τ= Time delay

we can obtain a range of values for each parameter to achieve a nominal test bench model and the extreme model.

The nominal values as well as the maximum and minimum allowable range for each parameter of the system (k, T_p and τ) are illustrated in Figures 5, 6 and 7 respectively. Additionally Table II shows the operating points selected for the nominal model and extreme model.

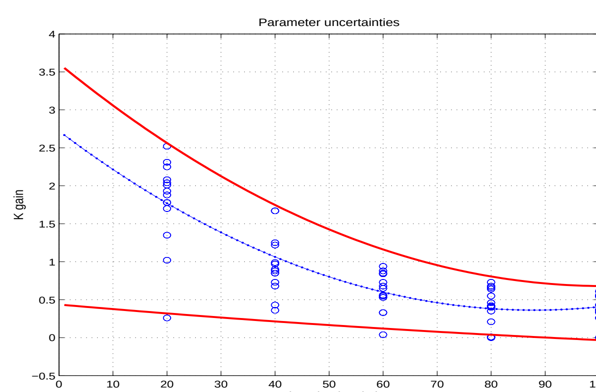

Fig. 5. *Parameter uncertainties* k

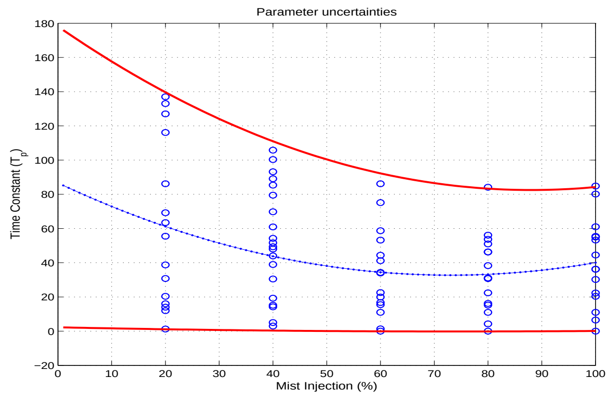

Fig. 6. *Parameter uncertainties T_p*

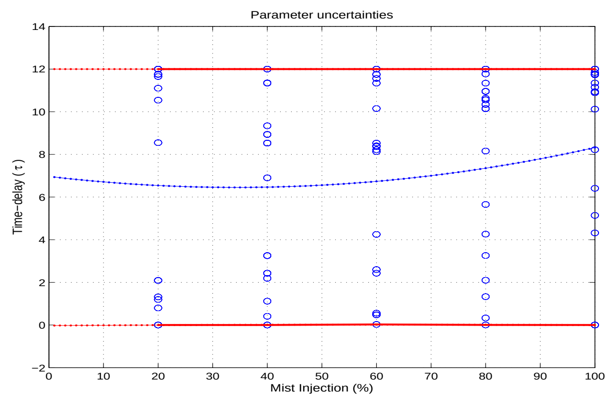

Fig. 7. *Parameter uncertainties τ*

TABLE II
PARAMETERS OF THE MODEL

	K gain	**Time constant (T_p)**	**Time-delay (τ)**
Nominal	1.2	40.32	8.3
Maximum	2.54	116.27	12
Minimum	0.6	6.25	5.13

Figure 8 shows a comparison between the proposed nominal model and the moisture dynamics of test bench when it is applied mist injection at 40%, resistance power at 20% and fan rotational velocity at 20%.

Remark 2: It is important to note that the gain decreases when the mist input increases independent of the variation of fan rotational velocity and resistance power. Also it is worth noting that the time constant (T_p) increases when the resistance power is less than fan rotational velocity. Finally, note that time-delay (τ) is larger when the fan rotational velocity works below 40%.

III. FORMULATION AND SOLUTION OF THE CONTROL PROBLEM

As seen in section II, the considered model is an input delay system with time-varying delay, since this kind of system has been considered in several studies science, in

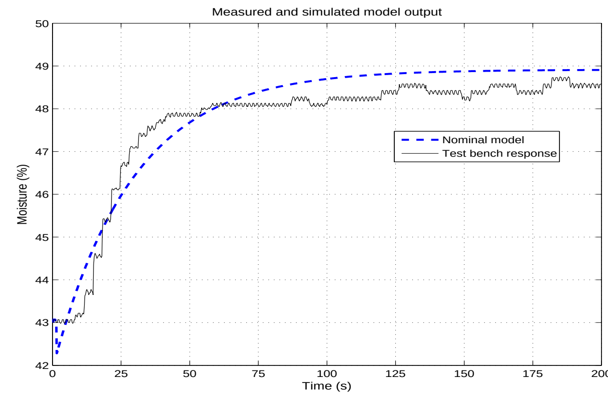

Fig. 8. *Nominal model vs Test bench*

closed-loop, the system becomes a state delayed system [19], [20], [21]. Then, the nominal system (1), can be represented in state space with uncertain parameters and time-varying input delay.

Now let consider a class of linear systems with time-varying delays and parameter uncertainties described by

$$(\Sigma): \quad \begin{aligned} \dot{x}(t) &= (\boldsymbol{A} + \boldsymbol{\Delta A}(t))\, x(t) \\ &\quad + (\boldsymbol{B} + \boldsymbol{\Delta B}(t))\, u\,(t - \tau(t)) \end{aligned} \tag{2}$$

$$x(t) = \phi(t), \quad \forall t \in [-h, 0] \tag{3}$$

where $x(t) \in \mathbb{R}^n$ is the state; $u(t) \in \mathbb{R}^m$ is the control input. $\boldsymbol{A}$, and $\boldsymbol{B}$ are known real constant matrices with appropriate dimensions. The scalar $h = \sup \tau(t)$ and $\phi(t)$ is a given differential initial function on $[-h, 0]$. $\boldsymbol{\Delta A}(t)$ and $\boldsymbol{\Delta B}(t)$ are unknown real matrices representing time-varying parametric uncertainties, which are assumed to be of the form

$$[\boldsymbol{\Delta A}(t) \quad \boldsymbol{\Delta B}(t)] = \boldsymbol{M}\boldsymbol{F}(t)\,[\boldsymbol{N}_1 \quad \boldsymbol{N}_2] \tag{4}$$

where $\boldsymbol{M}$, $\boldsymbol{N}_1$, and $\boldsymbol{N}_2$ are known constant matrices, and $\boldsymbol{F}(t) \in \mathbb{R}^{l \times q}$ is an unknown real time-varying matrix satisfying

$$\boldsymbol{F}(t)^T \boldsymbol{F}(t) \leq \boldsymbol{I} \tag{5}$$

$\boldsymbol{\Delta A}(t)$ and $\boldsymbol{\Delta B}(t)$ are said to be admisible if both (4) and (5) hold.

The time-varying input delay is assumed to be a continuous and bounded function satisfying for all 1 $t \geq 0$

$$0 < \tau(t) \leq h. \tag{6}$$

Following [17], the control objective is to minimize the following cost function

$$\boldsymbol{J} = \int_0^\infty \left[x(t)^T \boldsymbol{R}_1 x(t) + u(t)^T \boldsymbol{R}_2 u(t)\right] dt \tag{7}$$

where $\boldsymbol{R}_1 > 0$ and $\boldsymbol{R}_2 > 0$ are given constant matrices, considering the following linear state-feedback controller

$$u(t) = \boldsymbol{K}x(t), \qquad \boldsymbol{K} \in \mathbb{R}^{m\times n} \tag{8}$$

The guaranteed cost control problem to be addressed in this work is formulated as follows: given a scalar $h > 0$, design a state-feedback controller (8), such that for any time-varying delays $\tau(t)$ satisfying $0 < \tau(t) \leq h$, the closed-loop system (2) and (8) is asymptotically stable and the cost function in (7) has an upper bound for all admissible uncertainties. In this case, (8) is said to be a guaranteed cost state feedback controller.

Now, we are in a position to present a sufficient condition for the solvability of the guaranteed cost control problem under case of the test bench system. The result presented below has been presented in [17], and simplified to cope with the considered framework.

Theorem 1: Consider the uncertain time-delay system (Σ) and the cost function (7). Then, for given a scalar $h > 0$, the guaranteed cost control problem is solvable if there exist matrices $\boldsymbol{X} > 0$, $\boldsymbol{Z}_1 > 0$, $\boldsymbol{Z}_2 > 0$, $\boldsymbol{Y}$, $\boldsymbol{W}_1$, $\boldsymbol{W}_2$, $\boldsymbol{S}_1$, $\boldsymbol{S}_2$ and a scalar $\epsilon > 0$ such that the following matrix inequality holds

$$\begin{bmatrix} \boldsymbol{H}_1 & \boldsymbol{\Omega}_1 & \boldsymbol{\Omega}_2 & \boldsymbol{\Omega}_3 & \boldsymbol{\Omega}_4 \\ \boldsymbol{\Omega}_1^T & \boldsymbol{H}_2 & \boldsymbol{\Omega}_5 & \boldsymbol{\Omega}_6 & 0 \\ \boldsymbol{\Omega}_2^T & \boldsymbol{\Omega}_5^T & \boldsymbol{H}_3 & 0 & 0 \\ \boldsymbol{\Omega}_3^T & \boldsymbol{\Omega}_6^T & 0 & \boldsymbol{H}_4 & 0 \\ \boldsymbol{\Omega}_4^T & 0 & 0 & 0 & \boldsymbol{H}_5 \end{bmatrix} < 0 \tag{9}$$

where

$$\begin{aligned}
\boldsymbol{H}_1 &= \boldsymbol{X}\boldsymbol{A}^T + \boldsymbol{A}\boldsymbol{X} + \boldsymbol{B}\boldsymbol{Y} + \boldsymbol{Y}^T\boldsymbol{B}^T \\
&\quad -\boldsymbol{W}_1 - \boldsymbol{W}_1^T - \boldsymbol{S}_1 - \boldsymbol{S}_1^T + \epsilon\boldsymbol{M}\boldsymbol{M}^T \\
\boldsymbol{H}_2 &= \text{diag}\left(\boldsymbol{W}_2 + \boldsymbol{W}_2^T, \boldsymbol{S}_2 + \boldsymbol{S}_2^T\right) \\
\boldsymbol{H}_3 &= \text{diag}\left(0, h\boldsymbol{X}\boldsymbol{Z}_2^{-1}\boldsymbol{X}\right) \\
\boldsymbol{H}_4 &= \begin{bmatrix} \epsilon h^2\boldsymbol{M}\boldsymbol{M}^T - h\boldsymbol{Z}_2 & 0 \\ 0 & \epsilon\boldsymbol{I} \end{bmatrix} \\
\boldsymbol{H}_5 &= \text{diag}\left(-\boldsymbol{R}_1^{-1}, -\boldsymbol{R}_2^{-1}\right)
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{\Omega}_1 &= \begin{bmatrix} \boldsymbol{A}_1\boldsymbol{X} + \boldsymbol{W}_1 - \boldsymbol{W}_2^T & \boldsymbol{B}_1\boldsymbol{Y} + \boldsymbol{S}_1 - \boldsymbol{S}_2^T \end{bmatrix} \\
\boldsymbol{\Omega}_2 &= \begin{bmatrix} 0 & h\boldsymbol{S}_1 \end{bmatrix} \\
\boldsymbol{\Omega}_3 &= \begin{bmatrix} h\boldsymbol{X}\boldsymbol{A}^T + h\boldsymbol{Y}^T\boldsymbol{B}^T + \epsilon h\boldsymbol{M}\boldsymbol{M}^T & \boldsymbol{X}\boldsymbol{N}_1^T \end{bmatrix} \\
\boldsymbol{\Omega}_4 &= \begin{bmatrix} \boldsymbol{X} & \boldsymbol{Y}^T \end{bmatrix} \\
\boldsymbol{\Omega}_5 &= \text{diag}\left(0, h\boldsymbol{S}_2\right) \\
\boldsymbol{\Omega}_6 &= \begin{bmatrix} h\boldsymbol{X}\boldsymbol{A}_1^T & \boldsymbol{X}\boldsymbol{N}_1^T \\ h\boldsymbol{Y}^T\boldsymbol{B}_1^T & \boldsymbol{Y}^T\boldsymbol{N}_2^T \end{bmatrix}
\end{aligned}$$

In this case, a desired guaranteed cost state-feedback controller can be chosen as

$$u(t) = \boldsymbol{Y}\boldsymbol{X}^{-1}x(t) \tag{10}$$

and the corresponding cost function in (7) satisfies

$$\boldsymbol{J} \leq \phi(0)^T\boldsymbol{X}^{-1}\phi(0) + \int_{-h}^{0}\int_{\beta}^{0} \dot{\phi}(\alpha)^T\boldsymbol{Z}_2^{-1}\dot{\phi}(\alpha) \quad d\alpha \quad d\beta \tag{11}$$

Remark 3: Theorem 1 provides a sufficient condition for the solvability of the guaranteed cost control problem for the time-delay system (Σ). It is noted that the matrix inequality in (9) is not an LMI because of the term $\boldsymbol{X}\boldsymbol{Z}_2^{-1}$. In order to solve this non-convex problem, we introduce new variables $\boldsymbol{P}$ such that $\boldsymbol{X}\boldsymbol{Z}_2^{-1}\boldsymbol{X} \geq \boldsymbol{P}$. Then, we have

$$\boldsymbol{P}^{-1} \geq \boldsymbol{X}^{-1}\boldsymbol{Z}_2\boldsymbol{X}^{-1} \tag{12}$$

By Schur complement, it follows from (12) that

$$\begin{bmatrix} \boldsymbol{P}^{-1} & \boldsymbol{X}^{-1} \\ \boldsymbol{X}^{-1} & \boldsymbol{Z}_2^{-1} \end{bmatrix} \geq 0.$$

The following non-linear optimization problem can be stated, involving LMI conditions (for more details see [21]).

$$\text{minimise} \quad \text{tr}\left(\mathcal{P}\boldsymbol{P} + \mathcal{Z}_1\boldsymbol{Z}_1 + \mathcal{Z}_2\boldsymbol{P} + \mathcal{X}\boldsymbol{X}\right)$$

subject to

$$\begin{bmatrix} \boldsymbol{H}_1 & \boldsymbol{\Omega}_1 & \boldsymbol{\Omega}_2 & \boldsymbol{\Omega}_3 & \boldsymbol{\Omega}_4 \\ \boldsymbol{\Omega}_1^T & \boldsymbol{H}_2 & \boldsymbol{\Omega}_5 & \boldsymbol{\Omega}_6 & 0 \\ \boldsymbol{\Omega}_2^T & \boldsymbol{\Omega}_5^T & \mathcal{H}_3 & 0 & 0 \\ \boldsymbol{\Omega}_3^T & \boldsymbol{\Omega}_6^T & 0 & \boldsymbol{H}_4 & 0 \\ \boldsymbol{\Omega}_4^T & 0 & 0 & 0 & \boldsymbol{H}_5 \end{bmatrix} < 0 \tag{13}$$

where $\boldsymbol{H}_1$, $\boldsymbol{H}_2$, $\boldsymbol{H}_4$, $\boldsymbol{H}_5$, $\boldsymbol{\Omega}_i$, $i = 1, \ldots, 6$, are the same as those used for the Theorem 1, and

$$\mathcal{H}_3 = \text{diag}\left(0, h\boldsymbol{P}\right)$$

Using the Theorem 1, it can be seen that the guaranteed cost control problem is solvable and a desired guaranteed cost state-feedback controller can be obtained as in (10).

IV. APPLICATION TO THE TEST BENCH

A. Control design

In this section the application of the control strategy to the test bench is presented. In order to apply a guaranteed cost state-feedback control on the test bench system described in the previous section, the following model is considered

$$\boldsymbol{A} = \begin{bmatrix} -0.025 \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} 1.2 \end{bmatrix},$$

$$\boldsymbol{M} = [1],\ \boldsymbol{N_1} = [0.017],$$

$$\boldsymbol{N_2} = [1.34],$$

Note: the values of N_1 and N_2 are taken from the relationship between the maximum value and the nominal value of T_p and gain K respectively.

The time-delay $\tau(t)$ is assumed to satisfy (6), with

$$h = 12.$$

Now, using Theorem 1 and by (11), a guaranteed cost state-feedback control can be obtained with a maximum cost of $\boldsymbol{J} < 37.28$, but the tracking performance is not accurate enough.

In order to regulate the moisture inside of the tube in the test bench is possible to implement a state-feedback integral control. The aim is to regulate the output of the moisture $x(t)$ around a reference operating point profile (x_{ref}). In order to have a zero steady-state error $(x_{ref}(t) - x(t))$, an integrator is added to the system.

Now we can consider an extended system such that

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{E}(t) \end{bmatrix} &= \begin{bmatrix} A(t) & 0 \\ -I & -\lambda(t) \end{bmatrix} \begin{bmatrix} x(t) \\ E(t) \end{bmatrix} \\ &+ \begin{bmatrix} B(t) & 0 \\ 0 & x_{ref} \end{bmatrix} u(t-\tau) \end{aligned} \quad (14)$$

where E is the integral of the error. A new parameter $0 < \lambda(t) < \frac{1}{T_p}$ has been introduced as a "forgetting factor" for the integrator. The purpose of this term is to avoid high overshoots when changing the operating point by weighting down past accumulated errors. This parameter is designed to vanish in finite time.

Likewise, using the Theorem 1, a guarantee cost state-feedback integral control (GC-IC) is obtained as

$$u(t) = [-0.031 \quad 0.0005]x(t).$$

The corresponding closed-loop cost integral control satisfies $\boldsymbol{J} < 46.25$.

B. Simulation results

To illustrate the performance of state-feedback integral controller, the operation conditions are given by the nominal model. Then, Figure 9 illustrates how the time-varying input delay model can regulate the moisture around a moisture reference, which is given in 50%.

Similar results to those presented here could then be obtained using tuning of PI control by IMC (Internal Model Control) method [22]. The IMC-PI tuning rules have the advantage of using only a single tuning parameter to achieve a clear trade-off between closed-loop performance and robustness against model inaccuracies.

Consider the model of the test bench described by (1), and taking into account a PI controller given by

$$PI(s) = K_p(1 + \frac{1}{T_i s}), \quad (15)$$

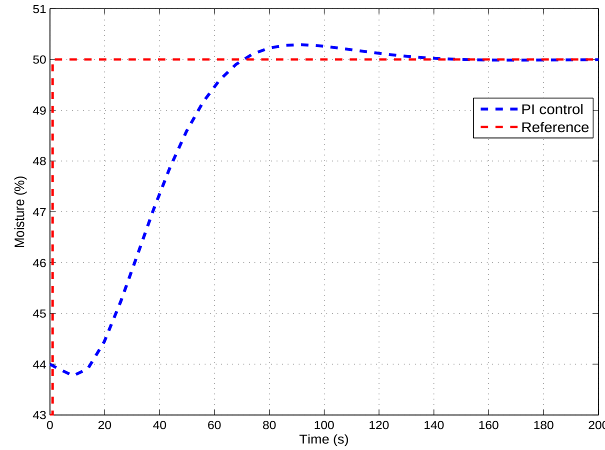

Fig. 9. *Response of the state-feedback integral control applied to nominal model*

The controller system (15) is able to do set-point tracking and disturbance rejection. The tuning rules IMC-PI for time-delayed system are given by

$$K_p = \frac{1}{K} \frac{T_p}{T_c + \tau},$$

and

$$T_i = \min\left(T_p, 4\left(T_c + \tau\right)\right)$$

where T_c is the trade-off output perf (small), usually $T_c = \tau$.

Considering the nominal model of the test bench described by

$$\frac{Moisture(\%)}{MistControl(\%)} = \frac{1.2}{40.32s + 1} e^{-8.3s}, \quad (16)$$

the values of the PI control are: $K_p = 2.03$ and $T_i = 0.025$.

Now, the GC-IC and IMC-PI controller are applied on the nominal model and extreme model, in order to compare the performance of the moisture. The responses are illustrated in the Figure 10.

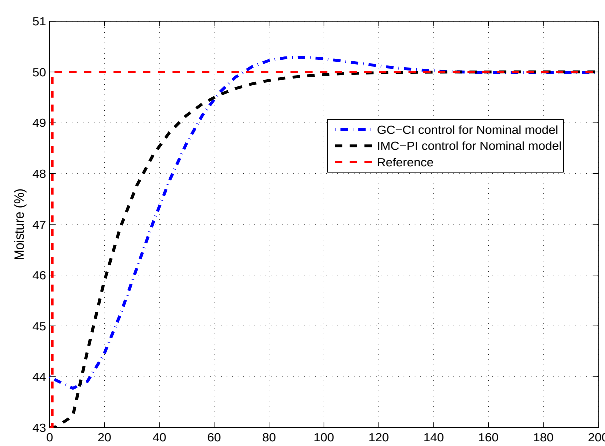

Fig. 10. *Comparison of IMC-PI control versus GC-IC*

In order to evaluate the performance of the strategies proposed (GC-IC and IMC-PI) on nominal and extreme models, we use the Integral Absolute Error (IAE) as an indicator of efficiency. The results shows that GC-IC has better performance on nominal model that in extreme models, while GC-IC improves its performance in the extreme models as we can see in Figure 11.

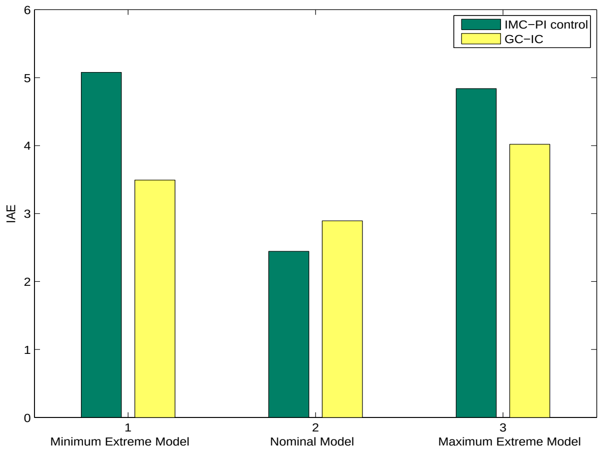

Fig. 11. *Performance indicator of IMC-PI control and GC-IC*

C. Experimental results

To prove the effectiveness of the proposed control strategies, IMC-PI and GC-IC controller are implemented into the test bench. As a first experiment the IMC-PI and GC-IC controller are applied on the test bench under following operating condition: the resistance control at 0% with the temperature in $26°C$ in initial condition and the fan rotational velocity in 20%. Figure 12 shows the behavior of the moisture control in order to achieve step-tracking given in 50% in the test bench.

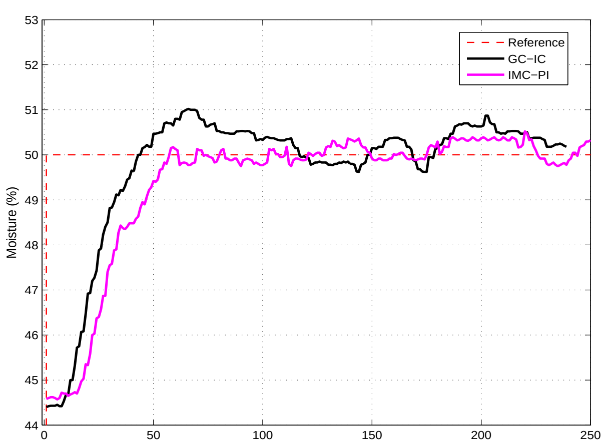

Fig. 12. *Experiment 1:Response of IMC-PI and GC-IC on the test bench*

In the second experiment is increases the resistance control at 40% and the fan rotational velocity at 40% with initial condition in temperature in $25°C$. In this case the moisture reference is the same that the previous experiment (50%). The experiment is show in Figure 13.

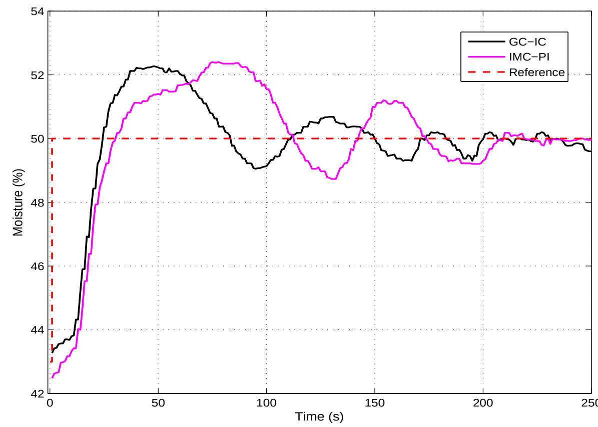

Fig. 13. *Experiment 2: Response of IMC-PI and GC-IC on the test bench*

The results shown in previous experiments illustrated that GC-IC has better performance on nominal model that in extreme models, while GC-IC improves its performance in the extreme models. Anyway, the controllers reach the step-tracking reference.

V. CONCLUSION

This paper has proposed a complete modeling and control study of moisture in a test bench. First a model was obtained from different experiments in order to identify the dynamics of moisture in the system. The purpose is to interpret the dynamics of moisture transport as a system with uncertain parameters and input delay (which is produced by the length of the tube) between the measured moisture and the mist injected. A guaranteed cost control is then proposed for the uncertain time-delay system. The state-feedback controller guarantees the robust stability of closed-loop system and an upper bound of the specific cost function for the maximum uncertainty in the test bench. Finally the efficiency of that control methodology is illustrated using simulation results and validated from experimental test.

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